The Theoretical Study of the Measuring Thermal Diffusivity of Semi-Infinite Solid Using the Photothermal Displacement

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A method of measuring the thermal diffusivity of semi-infinite solid material at room temperature using photothermal displacement is proposed. In previous works, within the constant thickness of material, the thermal diffusivity was determined by the magnitude and phase of deformation gradient as the relative position between the pump and probe beams. In this study, however, a complete theoretical treatment of the photothermal displacement technique has been performed for thermal diffusivity measurement in semi-infinite solid materials. The influence of parameters, such as, radius and modulation frequency of the pump beam and the thermal diffusivity, was studied. We propose a simple analysis method based on the zero-crossing position of real part of deformation gradient and the minimum position of phase as the relative position between two beams. It is independent of parameters such as power of pump beam, absorption coefficient, reflectivity, Poisson's ratio, and thermal expansion coefficient.

Key Words : Photothermal Displacement, Deformation Gradient, Phase, Thermal Diffusivity, Thermal Diffusion Length, Zero-Crossing Position, Minimum Position

Nomenclature

a : Radius of pump beam (μm)
α : Thermal diffusivity (cm^2/sec)
f : Modulation frequency (Hz)
α_{th} : Thermal expansion coefficient (1/K)
J_0, J_1 : 0th, 1st Bessel function
k : Thermal conductivity (W/m·K)
θ : Phase angle (degree)
λ : Absorption coefficient (1/m)
L_{th} : Thermal diffusion length (mm)
v : Poisson's ratio
P : Absorbed energy into specimen (W)
Q : Heat source (W)
ω : Angular frequency ($=2\pi f$) (Hz)
r : Relative position (mm)
T : Temperature (K)
u : Displacement vector (m)

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1. Introduction

The photothermal technique is a very useful tool for the research of new materials or the surface of materials, the measurement of thermo-physical properties for solid/liquid etc. As the measurement method of thermo-physical properties using the photothermal effect, a photothermal radiometry, photothermal refraction, photothermal deflection, and photothermal displacement are represented and much researches is actively being pursued.

The photothermal displacement method used in this study is a useful method to measure the magnitude and phase of the thermoelastic deformation gradient, which is produced by the absorbed light energy with a period on the surface of the material. In order to determine the thermal diffusivity of solid material, many experimental and theoretical studies have been conducted.

Olmstead et al. (1983) introduced a basic theory that had the pump beam modulated as a sine wave and the magnitude and phase of deformation gradient on the surface of material were calculated. They presented the possibility of thermal property measurement using photothermal displacement spectroscopy, based on comparison with experimental results. Li et al. (1991) calculated the magnitude of deformation at a point as they presented the analytical model with pump beam that was modulated into a square wave, and here, the characteristic frequency was defined as the modulation frequency when magnitude of deformation drops rapidly. They were unable to calculate the quantitative thermal property although they presented a simple equation used to obtain the thermal diffusivity with relation to the thermal diffusivity, the characteristic frequency, the thickness of material and the radius of pump beam. Applying the deformation gradient equation which was presented by Olmstead, Ogawa et al. (1999) determined the thermal diffusivity from relation between the phase of deformation gradient and the change of modulation frequency at a point. However, these methods have a relative large error and need much time to analyze the experimental results. Lee et al. (2000) proposed the equation used to determine the thermal diffusivity involving the relative distance that has a minimum value of phase curve with respect to the relative distance between pump beam and probe beam. As these studies were about the material with uniform thickness, these methods could not be applied to a thick material like a semi-infinite solid ; generally the thermal property of material that has thickness over 3 mm cannot be obtained theoretically. Therefore, in this study, in order to solve these problems, we present the analytical model for semi-infinite solid material, and the simple method applied to measure the thermal diffusivity was proposed using relations between the thermal diffusion length and relative distance when the real part of the deformation angle equals zero, the thermal diffusion length and relative position when the phase has a minimum value. Theoretical study was then performed and a simple analysis method was proposed from the numerical integration.

2. Principle and Theory

2.1 Principle

Figure 1 schematically shows the principle of the photothermal displacement method. The photothermal displacement method is based on detection of the displacement of the sample surface produced by the absorption of energy from a modulated light beam incident on the sample. The heating of the sample by the pump beam produces a temperature distribution and thermoelastic deformation of the sample, which can be detected by deflection of the probe beam reflected from the sample surface. And the information on the thermophysical properties of the sample can be obtained from measurements of deflection. Because thermoelastic deformation is changed by the thermal and optical properties, such as thermal diffusivity, absorption coefficient, etc., after ignoring the refraction by the air on the surface, the difference between the incidence and reflection angle is proportional to the deformation gradient on the sample surface.

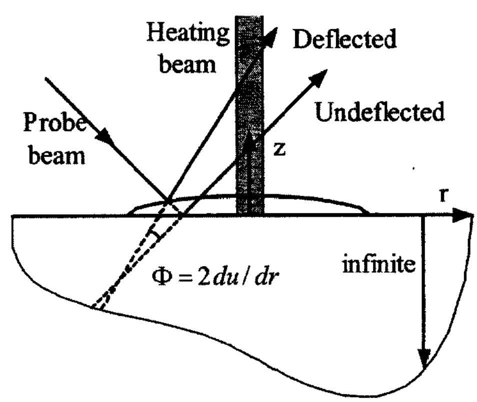


Fig. 1 The principle of the photothermal displacement method and theoretical model

Generally, there are two ways to determine thermal diffusivity through the photothermal displacement. The one is to use the deformation angle on the surface of the specimen and the other is by the use of the phase angle, which is produced by the phase lag as the position is far from the center of deformation or pump beam. Using the deformation gradient and phase method, the thermal diffusivity is determined by the comparison of the experimental and theoretical results.

2.2 Temperature Analysis

To obtain an expression for the photothermal deformation, we consider the homogeneous and isotropic semi-infinite solid with stress-free boundaries, no thermal conduction to the surrounding gas, and irradiated by a sinusoidally modulated laser beam, incident on the sample normal to the sample surface.

In order to find temperature distribution and deformation gradient of specimen, as in Fig. 1, a two-dimensional solid model that has a finite length in the direction of z and an infinite length in the direction of r is chosen. In temperature analysis, the conduction is considered as significant whereas convection and radiation are treated as negligible. With respect to each domain, the governing equation is the 2-D heat conduction equation having heat source in cylindrical coordinate :

$$\nabla^2 T_i - \frac{1}{\alpha_i}\frac{\partial T_i}{\partial t} = -\frac{1}{k_i}Q_i \quad (i=f, s) \tag{1}$$

where T is temperature, k is thermal conductivity, α is thermal diffusivity and Q is heat source produced by pump beam. Temperature and heat source are function of direction of radius (r), perpendicular direction (z) and time (t). The subscript f indicates the front gas of specimen and the subscript s represents the domain of specimen.

Heat flux and temperature are constant at $z=0$ and it is assumed that temperature is nearly zero because the thermal effect by pump beam can not have an influence on the specimen where z approaches an infinite. Also, as the temperature rise of the specimen by the pump beam is very small, the heat transfer by the convection or radiation is not considered. (Jeon et al., 2002 ; Carslaw and Jaeger, 1959) Therefore, the boundary conditions are

$$\begin{gathered} k_f\frac{\partial T_f}{\partial z}\bigg|_{z=0} = k_s\frac{\partial T_s}{\partial z}\bigg|_{z=0}, \\ T_f|_{z=0} = T_s|_{z=0}, \ \lim_{z\to\infty} T_f = \lim_{z\to-\infty} T_s = 0. \end{gathered} \tag{2}$$

The heat source is the pump beam which has a Gaussian intensity distribution and is controlled by the modulation frequency (f). The radius of the pump beam is considered to be the $1/e$ value of maximum intensity and the absorption coefficient (λ) is defined using the exponential law of light absorption. At the front gas region, light energy is not absorbed. Therefore the heat source is given by

$$Q_f=0, \ Q_s = \frac{P\lambda_s}{4\pi a^2}e^{-r^2/a^2+\lambda_s z}[1+\cos(\omega t)]. \tag{3}$$

Where the P represents an absorbed energy into specimen and is decided by the output power of pump beam and the reflectivity of specimen. The heat source $Q(r, z, t)$ is made of the independent term of time and the term that always oscillates with a constant frequency. The independent term of time is not considered since it does not have an effect on the phase of deformation and only the dependent term of time is considered. The analysis of the heat conduction equation is simplified by the transformation of a periodic function with time into steady state function. And in order to easily get the phase difference of deformation gradient, applying a complex method, the heat conduction equation, Eq. (3) is transformed into Eq. (4).

$$\begin{gathered} \nabla^2 \tilde{T}_f - \frac{i\omega}{\alpha_f}\tilde{T}_f = 0, \\ \nabla^2 \tilde{T}_s - \frac{i\omega}{\alpha_s}\tilde{T}_s = -\frac{P\lambda_x}{4\pi a^2 k_s}e^{-r^2/a^2+\lambda_s z} \end{gathered} \tag{4}$$

where $T_i(r, z, t) = \tilde{T}_i(r, z)\exp(\mathrm{i}\omega t)$.

Applying Hankel transforms to the differential equation that is transformed into steady state function, the equations are

$$\begin{gathered} \frac{\partial^2 t_f}{\partial z^2} - \xi_f^2 t_f = 0, \\ \frac{\partial^2 t_s}{\partial z^2} - \xi_s^2 t_s = -\frac{P\lambda_s}{8\pi k_s}e^{-\beta^2 r^2/a^2+\lambda z} \end{gathered} \tag{5}$$

where $t_i(\beta, z) = \int_0^\infty \tilde{T}_i(r, z)\, rJ_0(\beta r)\, dr$, $\xi_{f/s} = \sqrt{\beta^2 + i\omega/a_{f/s}}$.

The boundary conditions which are transformed by the complex method and the Hankel transforms are

$$k_f \frac{\partial t_f}{\partial z}\bigg|_{z=0} = k_s \frac{\partial t_s}{\partial z}\bigg|_{z=0}, \quad t_f|_{z=0} = t_s|_{z=0}, \lim_{z\to\infty} t_f = \lim_{z\to-\infty} t_s = 0. \tag{6}$$

The Eq. (5) is the 1st-order differential equation and the solution is written as Eq. (7). In addition, the coefficients A, B, C and D are obtained from the transformed boundary conditions, Eq. (6). The t_{sp} is particular solution of t_s and is inserted into the Eq. (5) and then, the unknown F is obtained.

$$t_f = Ae^{-\xi_f z} + Be^{\xi_f z}, \quad t_s = Ce^{-\xi_s z} + De^{\xi_s z} + t_{sp},\ t_{sp} = Fe^{\lambda_s z} \tag{7}$$

where

$$F = -\frac{P\lambda_s}{8\pi k_s}\frac{e^{-\beta^2 a^2/4}}{\delta^2 + \lambda_s^2},\ A = \frac{k_s\delta_s - k_s\lambda_s}{k_f\delta_f + k_s\delta_s}F, \quad B = C = 0,\ D = \frac{k_s\lambda_s + k_f\delta_f}{k_f\delta_f + k_s\delta_s}F. \tag{8}$$

Inserting the coefficients A, D, C, D and F into Eq. (7) and using Inverse Hankel transforms and complex method, the equations of the final temperature distribution are obtained. The equations can be expressed as

$$\tilde{T}_i(r, z) = \int_0^\infty t_i(\beta, z)\, \beta J_0(\beta r)\, d\beta \tag{9}$$

$$T_f(r, z, t) = \frac{P\lambda_s}{8\pi k_s}\int_0^\infty \beta d\beta J_0(\beta r) \left(\frac{k_s\delta_s - k_s\lambda_s}{k_f\delta_f + k_s\delta_s}\right)\left(\frac{e^{-\beta^2 a^2/4}}{\delta_s^2 - \lambda_s^2}\right)e^{-\delta_f z}e^{i\omega t} \quad T_s(r, z, t) = \frac{P\lambda_s}{8\pi k_s}\int_0^\infty \beta d\beta J_0(\beta r) \left(e^{\lambda_s} - \frac{k_s\lambda_s + k_s\delta_s}{k_f\delta_f + k_s\delta_s}e^{\delta_s}\right)\left(\frac{e^{-\beta^2 a^2/4}}{\delta_s^2 - \lambda_s^2}\right)e^{i\omega t}. \tag{10}$$

2.3 Thermoelastic analysis

The result of temperature analysis is applied to the thermo elastic model using the result of temperature analysis. The equation used here is the thermoelastic equation (Navier Equation) as Eq. (11). Assuming that external force does not act on the periphery of the specimen (Nowachi, 1986), the boundary conditions are expressed as

$$\nabla^2\vec{u} + \frac{\nabla(\nabla\cdot\vec{u})}{1-2v} = \frac{2(1+v)}{1-2v}a_{th}\nabla T \tag{11}$$

$$\sigma_{rz}|_{z=0} = 0,\ \sigma_{zz}|_{z=0} = 0,\ \lim_{z\to-\infty}\vec{u} = 0. \tag{12}$$

The deformation of specimen (u) which is caused by the change of temperature is expressed as the sum of the particular solution (u_p) and homogeneous solution (u_h) as Eq. (13). In addition, the particular solution can be expressed as the thermoelastic potential function(ϕ). Inserting this equation into Eq. (11), the solutions are divided into the particular and homogeneous solution as the Eqs. (14) and (15). The homogeneous solution is transformed into the form of the bi-harmonic equation as the Eq. (17) using Love function expressed as the Eq. (16). Therefore, the expression with the deformation of specimen is given by Eq. (18), and the perpendicular deformation which should be considered in this work is written as Eq. (19). Also, the boundary conditions are expressed as Eq. (20) applying the thermoelastic potential function and Love function.

$$\vec{u} = \vec{u}_p + \vec{u}_h = \nabla\phi + \vec{u}_h \tag{13}$$

$$\nabla^2\phi = \frac{1+v}{1-v}a_{th}T \tag{14}$$

$$(1-2v)\nabla^2\vec{u}_h - \nabla(\nabla\cdot\vec{u}_h) = 0 \tag{15}$$

$$\vec{u}_h = \frac{2(1-v)\nabla^2\psi - \nabla(\nabla\cdot\psi)}{1-2v} \tag{16}$$

$$\nabla^4\psi = 0 \tag{17}$$

$$\vec{u} = \nabla\phi + \frac{2(1-v)\nabla^2\psi - \nabla(\nabla\cdot\psi)}{1-2v} \tag{18}$$

$$u_z = \frac{\partial\phi}{\partial z} + \frac{(1-v)\nabla^2\psi - \partial^2\psi/\partial z^2}{1-2v} \tag{19}$$

$$(1-2v)\frac{\partial\phi}{\partial z} + (1-v)\nabla^2\psi - \frac{\partial^2\psi}{\partial z^2} = 0, \quad (1-2v)\left(\frac{\partial^2\phi}{\partial z^2} - \nabla^2\phi\right) + \frac{\partial}{\partial z}\left[(2-v)\nabla^2\psi - \frac{\partial^2\psi}{\partial z^2}\right] = 0 \tag{20}$$

The ϕ and ψ are a function of r and z and in order to analyze easily, these equations are applied to the Hankel transforms. And then, these are transformed into the only functions of z. The transformed equations are

$$\frac{\partial^2\tilde{\phi}}{\partial z^2}-\beta^2\tilde{\phi}=\frac{1+v}{1-v}\alpha_{th}t_s \tag{21}$$

$$\frac{\partial^4\tilde{\psi}}{\partial z^4}-2\beta^2\frac{\partial^2\tilde{\psi}}{\partial z^2}+\beta^2\tilde{\psi}=0 \tag{22}$$

where

$$\tilde{\phi}(\beta, z)=\int_0^\infty \phi(r, z)\, rJ_0(\beta r)\, dr,$$

$$\tilde{\psi}(\beta, z)=\int_0^\infty \psi(r, z)\, rJ_0(\beta r)\, dr.$$

In addition, using the Hankel transforms, the boundary conditions and deformation(u) are expressed as

$$\begin{aligned}&(1-2v)\frac{\partial\tilde{\phi}}{\partial z}+(1-v)\nabla^2\tilde{\psi}-\frac{\partial^2\tilde{\psi}}{\partial z^2}=0\\ &(1-2v)\beta^2\tilde{\phi}-\beta^2(2-v)\frac{\partial\tilde{\psi}}{\partial z}-(1-v)\frac{\partial^3\tilde{\psi}}{\partial z^3}=0\end{aligned} \tag{23}$$

$$\tilde{u}_z=\frac{\partial\tilde{\phi}}{\partial z}+\frac{\partial^2\tilde{\psi}}{\partial z^2}-\frac{2(1-v)\beta^2\partial\tilde{\psi}}{1-2v} \tag{24}$$

where

$$\tilde{u}_z(\beta, z)=\int_0^\infty u(r, z)\, rJ_0(\beta r)\, dr.$$

The solutions of differential equations that are expressed in Eqs. (21) and (22) can be written as Eqs. (25) and (26) and the unknowns are obtained by applying boundary conditions. Here, $\tilde{\phi}_p$ which is the particular solution of transformed potential function, can be represented as Eq. (27) and the unknowns are obtained by inserting into Eq. (21).

$$\tilde{\phi}(r, z)=Me^{-\beta z}+Ne^{\beta z}+\tilde{\phi}_p \tag{25}$$

$$\tilde{\psi}(r, z)=(A+Bz)e^{-\beta z}+(C+Dz)e^{\beta z} \tag{26}$$

where

$$\begin{aligned}\tilde{\phi}_p&=Ge^{\lambda_s z}+He^{\delta_s z}.\\ &=\frac{1+v}{1-v}\frac{\alpha_{th}P\lambda_s}{4\pi k_s}\frac{e^{-\beta^2a^2/4}}{\delta_s^2-\lambda_s^2}\\ &\left(\frac{e^{\lambda_s z}}{\lambda_s^2-\beta^2}-\frac{(k_s\lambda_s+k_f\delta_f)e^{\delta_s z}}{(\delta_s^2-\beta^2)(k_f\delta_f+k_s\delta_s)}\right).\end{aligned} \tag{27}$$

Applying the boundary condition of Eq. (12), it is proper that unknowns M, A and B equal zero. Inserting the term that is related to homogeneous solution of Eq. (26) into boundary condition of Eq. (23), unknown C and D can be obtained as the function of $\tilde{\phi}$. Therefore, the Eq. (28) is formulated from Eq. (24), and the equation of surface which is used in this work is written as Eq. (29). Here, the unknown N has no connection with the value of Eq. (29).

$$\tilde{u}_z=\frac{\partial\tilde{\phi}}{\partial z}+2\beta De^{\beta z}-\frac{\beta^2(C+Dz)e^{\beta z}}{1-2v} \tag{28}$$

$$\begin{aligned}\tilde{u}_z|_{z=0}&=\frac{\partial\tilde{\phi}}{\partial z}+2\beta D-\frac{\beta^2C}{1-2v}\\ &=2(1-v)\left[\frac{\partial\tilde{\phi}}{\partial z}-\beta\tilde{\phi}\right]\end{aligned} \tag{29}$$

where

$$C=(1-2v)\left[\frac{1-2v}{\beta^2}\frac{\partial\tilde{\phi}}{\partial z}+\frac{2v}{\beta}\tilde{\phi}\right],$$

$$D=(1-2v)\left[\frac{1}{\beta}\frac{\partial\tilde{\phi}}{\partial z}-\tilde{\phi}\right].$$

Through the Inverse Hankel Transforms, the perpendicular deformation at the surface is expressed as

$$u_z|_{z=0}=\frac{(1+v)\alpha_{th}P\lambda_s}{4\pi k_s}\int_0^\infty \beta^2 d\beta J_0(\beta r)\left\{\frac{e^{-\beta^2a^2/4}}{\delta_s^2-\lambda_s^2}\left(\frac{1}{\lambda_s+\beta}-\frac{k_s\lambda_s+k_f\delta_f}{(\delta_s+\beta)(k_f\delta_f+k_s\delta_s)}\right)\right\}$$

Differentiating the above equation with respect to radius(r), the final deformation gradient of specimen at the surface can be written as

$$\left.\frac{du_z}{dr}\right|_{z=0}=-\frac{(1+v)\alpha_{th}P\lambda_s}{4\pi k_s}\int_0^\infty\left\{\frac{e^{-\beta^2a^2/4}}{\delta_s^2-\lambda_s^2}\left(\frac{1}{\lambda_s+\beta}-\frac{k_s\lambda_s+k_f\delta_f}{(\delta_s+\beta)(k_f\delta_f+k_s\delta_s)}\right)\beta^2J_1(\beta r)\right\}d\beta \tag{30}$$

The deformation gradient of the specimen at the surface is largely influenced by the relative position, modulation frequency, radius of pump beam and thermal diffusivity. In addition, the Poisson ratio, coefficient of thermal expansion and power of pump beam in the first term of Eq. (30) play a part in the constants that increase or decrease the amplitude of deformation gradient.

3. Results and Analysis

Figure 2 shows the typical plot of temperature distribution at the surface. There is some difference according to the analysis condition such as thermal diffusivity, thermal conductivity, thermal expansion coefficient, reflectivity, and optical absorption coefficient, but the maximum difference of temperature is about 1~3K usually. Because the temperature increment is much small, the measured value in this study could be the thermal diffusivity of room temperature.

As Fig. 3 is shows, the representative signals obtained by photothermal displacement method are the magnitude and phase of deformation gradient. As the relative position between the pump and probe beam increases, the real part of deformation gradient increases sharply to the near of radius of pump beam, but after a maximum point, it decreases gradually. The phase of deformation shows a decline until a position and increases to the convergence of a temporary value. In Fig. 3, only one side from the center of deformation is displayed, but in fact the other side has a symmetrical curve. The scaling of the height of the deformation gradient and real part of deformation gradient curve were adjusted arbitrarily.

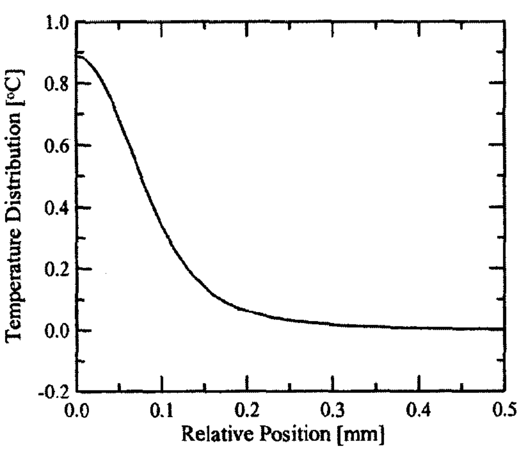


Fig. 2 Plot of temperature rise as a function of the distance r from the beam center. Parameters : using the thermal and optical properties of pure iron, $a=80\mu m$, $P=0.12W$, $f=500Hz$.

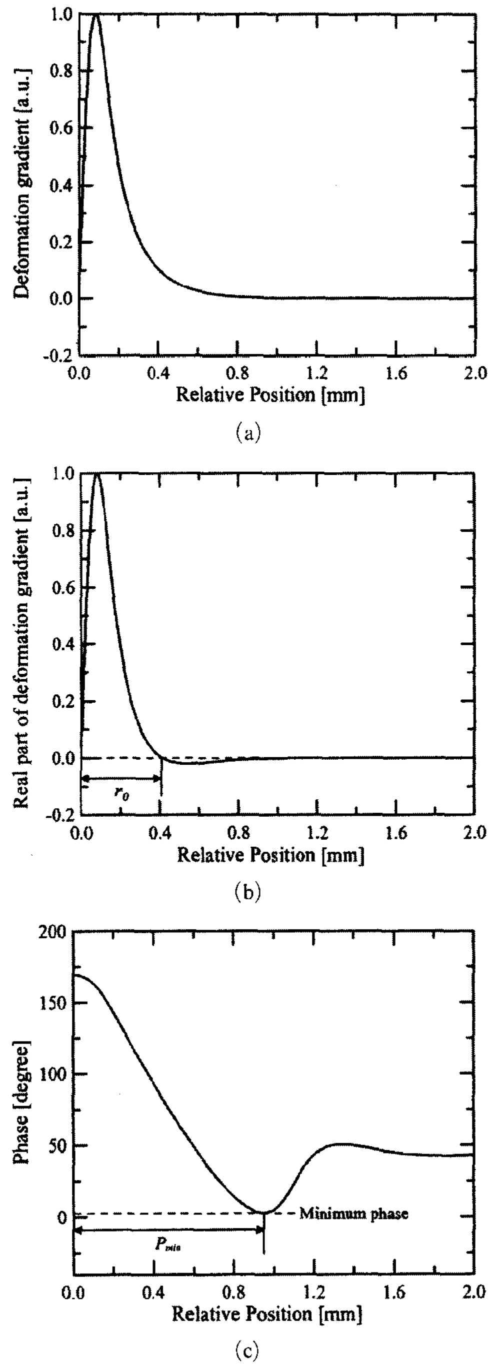


Fig. 3 Real part of deformation gradient and phase as a function of relative position. (a) Deformation gradient (b) Real part of deformation gradient (c) Phase

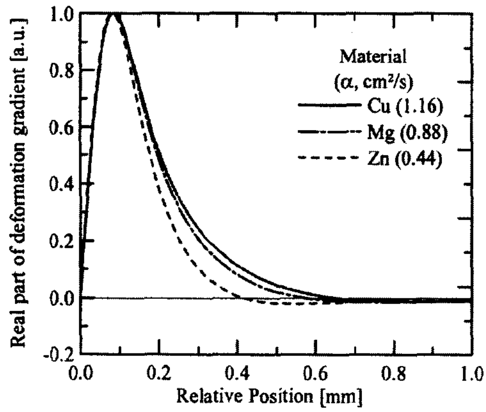
The phase decreases up to a certain point, then starts to increase and approaches an asymptotic value. This has the position where the phase has a minimum value. The position p_{min} is defined as the "minimum position". The real part of deformation gradient is similar to the deformation gradient, but this has the position where the real part of deformation gradient is zero. The position r_0 is defined as the "zero-crossing position". The minimum position and zero-crossing position are a function of the thermal diffusion length and the radius of pump beam. Especially, the thermal diffusion length exerts the highest effect than other ingredients. The methods of determination of thermal diffusivity using a zero-crossing position and minimum position is very simple because there is not any more normalizing work in the process of the analytical comparisons between the experimental and theoretical value.

3.1 The influence of parameters

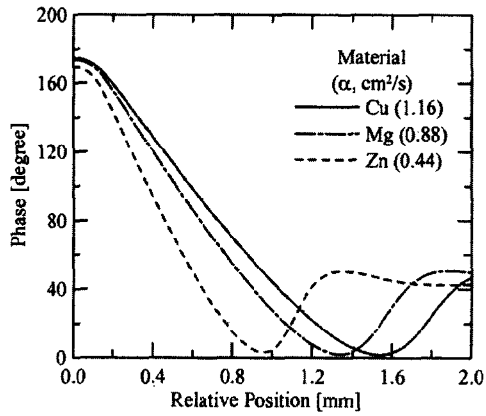
The influence of parameters such as modulation frequency, thermal diffusivity, and pump beam diameter on the minimum position and zero-crossing position was analytically studied.

As the various relative position between the pump and probe beam changes, Fig. 4 shows the calculated result ; that is the real part of deformation gradient about the materials which have different thermal diffusivity one another, where the radius of pump beam is 80μm and modulation frequency is 500Hz. As the thermal diffusivity of materials is high, Both the zero-crossing position of real part of deformation gradient and the minimum position of phase become more distant from the center of pump beam as shown in Fig. 4 (a) and (b) respectively. These results are caused by the fact that the absorbed energy in the material of which thermal diffusivity is high, diffuses over a larger area than materials whose thermal diffusivity is relatively low.

The Fig. 5 shows the calculated real part of deformation gradient and phase angle to the radial direction when the modulation frequency of pump beam is 500Hz, 1kHz and 2kHz. The radius of pump beam is 80μm with the thermal and optical properties of pure copper. As the modulation frequency is high, both the zero-



(a)



(b)

Fig. 4 Real part of deformation gradient and phase as a function of relative position for different samples. (a) Real part of deformation gradient (b) Phase

crossing position and the minimum position become closer to the center of pump beam. And the minimal value of phase angle becomes smaller, because the absorbed energy in the material per period reduces as the modulation frequency increases. So, the absorbed energy relatively diffuses small area.

To see the influence of the pump beam radius, Fig. 6 represents the calculated the real part and the phase of deformation gradient when the radius is 100, 150 and 200μm with the thermal and optical properties of pure magnesium in sequence. If the radius is increased, the directly heated area is increased and the thermally diffused area is

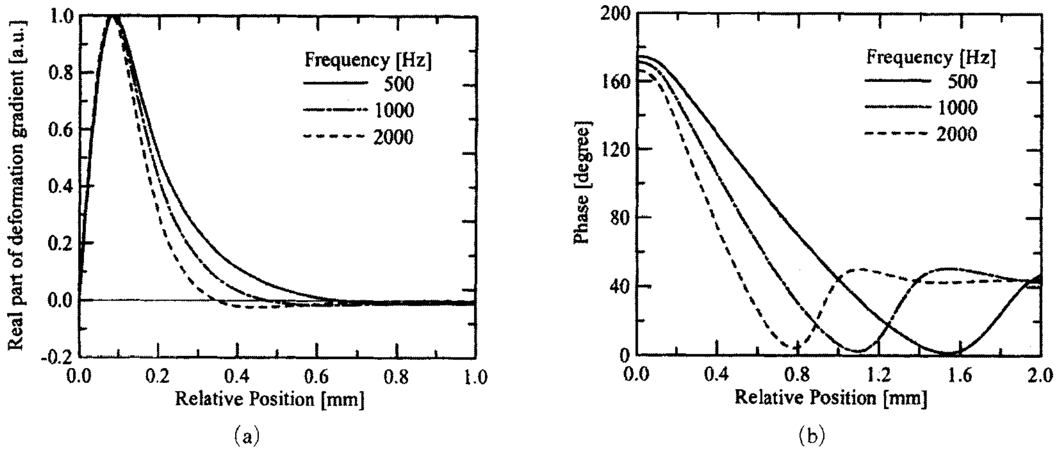


Fig. 5 Real part of deformation gradient and phase as a function of relative position for various frequency of pump beam. (a) Real part of deformation gradient (b) Phase

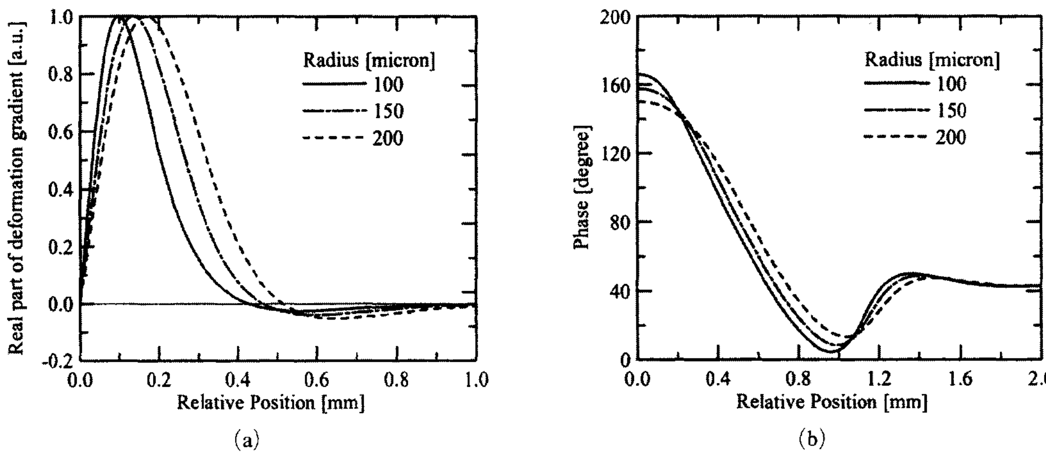


Fig. 6 Real part of deformation gradient and phase as a function of relative position for various pump beam radius. (a) Real part of deformation gradient (b) Phase

increased as well. Consequently, the zero-crossing position becomes more distant from the center of pump beam and the minimum point of phase angle becomes distant as well. But the difference between the maximum and minimum phase angle is reduced and the gradient of the section where the phase angle declines rapidly is barely changed.

3.2 Determination of thermal diffusivity

In order to determine thermal diffusivity through a photothermal displacement, there are a few methods that minimize the error between the experimental and theoretical phase angle or deformation curve with a variable thermal diffusivity at the characteristic frequency. But these methods have a relatively large error and need much time to analyze the experimental results.

In this study, it is known that the zero-crossing position only depends on the thermal diffusivity, the pump beam radius and the modulation frequency. So considering this characteristic relation, as shown in Fig. 7, the simple relation equation is obtained at the fixed pump beam radius. There is a linear relation between the zero-crossing position and thermal diffusion length $(=(\alpha/\pi f)^{1/2})$ that is decided by the thermal diffusivity and the modulation frequency like Eq. (31). Therefore, the zero-crossing position is

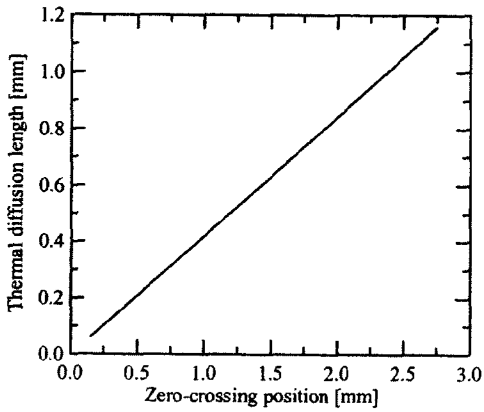


Fig. 7 Zero-crossing position versus thermal diffusion length at pump beam radius $a=60\mu m$.

found from this relation and the thermal diffusivity can be easily obtained from it.

$$L_{th}=C_1 r_0+C_2 \tag{31}$$

Like Eq. (32), thermal diffusivity is made of a simple equation from the definition of thermal diffusion length and Eq. (31)

$$\alpha=\pi f[C_1 r_0+C_2]^2 \tag{32}$$

where the pump beam radius is $60\mu m$, C_1 and C_2 are 0.423 and -4.473×10^{-3} respectively.

In Fig. 8, the thermal diffusion length (L_{th}) is plotted versus the minimum (p_{min}). When the radius of the pump beam is $60\mu m$, the best fitting relation between thermal diffusion length and

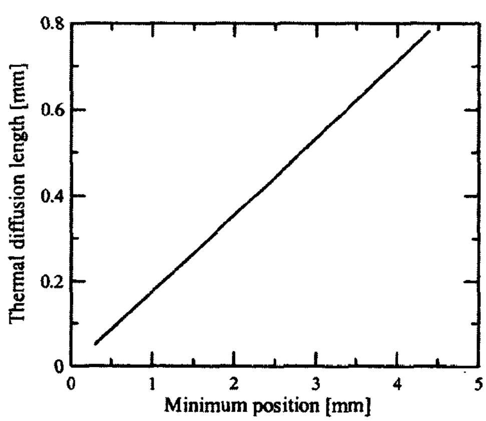


Fig. 8 Minimum phase position versus thermal diffusion length at pump beam radius $a=60\mu m$.

minimum position is form of the linear expression as follows

$$L_{th}=A_1 r_0+A_2 \tag{33}$$

where A_1 and A_2 are 0.178 and -2.25×10^{-3} respectively. From the definition of thermal diffusion length, the thermal diffusivity is determined easily and simply. For the known sample thickness and the radius of the pump beam, therefore, the thermal diffusivity is determined by the measurement of the minimum position.

4. Conclusions

In this study, to apply the photothermal displacement method to the infinite solid, theoretical analysis is performed. To measure the thermal diffusivity easily and precisely, the relation among the thermal diffusivity, the radius of pump beam and the modulation frequency that affect to the phase and the real part of deformation gradient, is analyzed. The following conclusion are obtained :

(1) As the thermal diffusivity increases, r_0 becomes more distant from the center of pump beam and the minimum point of phase angle becomes more distant as well.

(2) As the modulation frequency increases, r_0 becomes closer to the center of pump beam and the relative position where phase is to be minimized becomes closer also. The minimum value of phase also becomes smaller.

(3) As the pump beam radius increases, r_0 becomes more distant from the center of pump beam and the relative position to be minimum phase angle becomes more distant. On the other hand, the difference between the maximum and minimum phase angle is reduced and the gradient of the section where the phase angle declines rapidly, is barely changed.

(4) The relation between the thermal diffusion length and the minimum position is linearly independent with materials. And the zero-crossing position (r_0) is linear to the thermal diffusion length. Applying these methods, it is very simple and easy to determine the thermal diffusivity with accuracy. The new equation is independent of the

output power of pump beam, absorption coefficient, reflectivity, Poisson's ratio and thermal expansion coefficient.

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